10/8/2020 B.Sc. Part II 4th Paper

DIFFERENTIAL EQUATIONS

LDECC (contd.)

* When RHS is of the form $e^{ax}.V$ where V is any function of x.

So, formula to find PI

$$\boxed{\frac{1}{f(D)} e^{ax}.V = e^{ax}\frac{1}{f(D+a)}.V}$$

Q. Solve $(D^2-2D+1)y = x^2 e^{3x}$

Soln. For CF, $(D-1)^2 = 0 \Rightarrow D = 1, 1$

$\therefore CF = (C_1 + C_2 x) e^{x}$

For PI, $PI = \frac{1}{(D^2-2D+1)} e^{3x}.x^2$

$\Rightarrow PI = \frac{1}{(D-1)^2} e^{3x}.x^2$

$= e^{3x} \frac{1}{[(D+3)-1]^2} x^2$

$$\Rightarrow PI = e^{3x} \cdot \frac{1}{(D+2)^2} x^2 = e^{3x} \cdot \frac{1}{2^2\left(\frac{D}{2}+1\right)^2} x^2$$

$$\Rightarrow PI = \frac{e^{3x}}{4} \cdot \frac{1}{\left(1+\frac{D}{2}\right)^2} x^2 = \frac{e^{3x}}{4} \left(1+\frac{D}{2}\right)^{-2} x^2$$

$$\Rightarrow PI = \frac{e^{3x}}{4} \left[1 - \frac{D}{2} + \frac{3}{4} D^2 - - - -\right] x^2$$

$$\Rightarrow PI = \frac{e^{3x}}{4} \left[x^2 - \frac{D(x^2)}{2} + \frac{3}{4} D^2(x^2) + 0\right]$$

$$\Rightarrow PI = \frac{e^{3x}}{4} \left[x^2 - \frac{2x}{2} + \frac{3}{4} \cdot 2\right]$$

$$\Rightarrow PI = \frac{e^{3x}}{4} \left[x^2 - x + \frac{3}{2}\right]$$

$$\Rightarrow PI = \frac{e^{3x}}{8} (2x^2 - 2x + 3).$$

Hence, complete solution is given by

$$y = CF + PI$$

$$\Rightarrow y = (c_1 + c_2 x) e^{x} + \frac{e^{3x}}{8} (2x^2 - 2x + 3).$$